ELECTROMAGNETIC LEVITATION SYSTEM

Mathematical Model

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web: http://zeltom.com

email: info@zeltom.com

47001 Harbour Pointe Ct.

Belleville, MI 48111

USA

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1. INTRODUCTION

The electromagnetic levitation system controls the magnetic field generated by an electromagnet to levitate a small permanent magnet in midair. With an appropriate controller in the loop, the small magnet levitates in the air indefinitely without any disturbance. The vertical position of the levitating magnet is measured using a linear Hall effect sensor and the current in the electromagnet is actively controlled to achieve stable levitation.

The electromagnetic levitation system is one of the classical experimental setups to illustrate some of the analysis and design methods in control system education. It is also used to evaluate novel analysis and design methods in control system research. The system is highly nonlinear, open-loop unstable and extremely challenging to control robustly.

The electromagnetic levitation system has been developed to offer a low-cost experimental demonstration setup for students, educators and researchers. The developed system is highly efficient, and works with both the disc magnet and sphere magnet. The model presented below is the model of the system with the disc magnet.

2. SYSTEM MODEL

The model of the electromagnetic levitation system is shown in Figure 1, where R is the resistance of the coil, L is the inductance of the coil, v is the voltage across the electromagnet, i is the current through the electromagnet, m is the mass of the levitating magnet, g is the acceleration due to gravity, d is the vertical position of the levitating magnet measured from the bottom of the coil, f is the force on the levitating magnet generated by the electromagnet and e is the voltage across the Hall effect sensor.

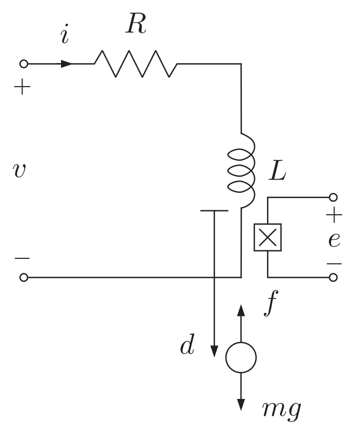

Figure 1. Electromagnetic levitation system model.

The force applied by the electromagnet on the levitating magnet can be closely approximated as

$$f = k\frac{i}{d^4}, \tag{2.1}$$

where k is a constant that depends on the geometry of the system [1]. The voltage across the Hall effect sensor induced by the levitating magnet and the coil can be closely approximated as

$$e = \alpha + \beta\frac{1}{d^2} + \gamma i + n, \tag{2.2}$$

where α, β and γ are constants that depend on the Hall effect sensor used as well as the geometry of the system and n is the noise process that corrupts the measurement [2]. It follows from Newton's second law that

$$m\ddot{d} = mg - k\frac{i}{d^4}. \tag{2.3}$$

Moreover, it follows from the Kirchhoff's voltage law that

$$v = Ri + L\dot{i}. \tag{2.4}$$

Letting $x = \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix} = \begin{bmatrix} d & \dot{d} & i \end{bmatrix}$ be the state of the system, $z = d$ be the controlled output, $y = e$ be the measured output, $u = v$ be the control input and $w = n$ be the disturbance/noise input, the standard state equation description of the system can be written as

$$
\begin{aligned}
\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} &= \begin{bmatrix} x_2 \\ -\frac{k}{m}\frac{x_3}{x_1^4} + g \\ -\frac{R}{L}x_3 + \frac{1}{L}u \end{bmatrix}, \\
z &= x_1, \\
y &= \beta\frac{1}{x_1^2} + \gamma x_3 + \alpha + w.
\end{aligned}
\tag{2.5}
$$

The equilibrium points of the system are at

$$
\begin{bmatrix} x_{1e} \\ x_{2e} \\ x_{3e} \end{bmatrix} = \begin{bmatrix} \pm(\frac{ku_e}{gmR})^{1/4} \\ 0 \\ \frac{u_e}{R} \end{bmatrix}, \tag{2.6}
$$

where u_e is the required equilibrium coil voltage to suspend the levitating magnet at $x_{1e} = \pm d_e$. Clearly, the equilibrium point that is of interest is the one with the positive sign.

The Jacobian linearization of the system about the equilibrium point is

$$
\begin{aligned}
\delta\dot{x} &= A\delta x + B_1\delta w + B_2\delta u, \\
\delta z &= C_1\delta x + D_{11}\delta w + D_{12}\delta u, \\
\delta y &= C_2\delta x + D_{21}\delta w + D_{22}\delta u,
\end{aligned}
\tag{2.7}
$$

where $\delta x = x - x_e$, $\delta w = w - w_e$, $\delta u = u - u_e$, $\delta z = z - z_e$, $\delta y = y - y_e$ and

$$
\begin{aligned}
A &= \begin{bmatrix} 0 & 1 & 0 \\ \frac{4g(gmR)^{1/4}}{(ku_e)^{1/4}} & 0 & -\frac{gR}{u_e} \\ 0 & 0 & -\frac{R}{L} \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{L} \end{bmatrix}, \\
C_1 &= \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}, \qquad D_{11} = \begin{bmatrix} 0 \end{bmatrix}, \quad D_{12} = \begin{bmatrix} 0 \end{bmatrix}, \\
C_2 &= \begin{bmatrix} -\frac{2\beta(gmR)^{3/4}}{(ku_e)^{3/4}} & 0 & \gamma \end{bmatrix}, \quad D_{21} = \begin{bmatrix} 1 \end{bmatrix}, \quad D_{22} = \begin{bmatrix} 0 \end{bmatrix}.
\end{aligned}
\tag{2.8}
$$

Note that $w_e = 0$, $z_e = x_{1e}$ and $y_e = \beta/{x_{1e}}^2 + \gamma x_{3e} + \alpha$.

The transfer matrix of the linearized system is

$$P(s) = \begin{bmatrix} 0 & \dfrac{-\dfrac{gR}{u_e L}}{(s+\dfrac{R}{L})[s^2 - \dfrac{4g(gmR)^{1/4}}{(ku_e)^{1/4}}]} \\ 1 & \dfrac{\dfrac{\gamma}{L}s^2 + [\dfrac{2\beta gR(gmR)^{3/4}}{Lu_e(ku_e)^{3/4}} - \dfrac{4\gamma g(gmR)^{1/4}}{L(ku_e)^{1/4}}]}{(s+\dfrac{R}{L})[s^2 - \dfrac{4g(gmR)^{1/4}}{(ku_e)^{1/4}}]} \end{bmatrix}. \tag{2.9}$$

Note that

$$\begin{bmatrix} \Delta Z(s) \\ \Delta Y(s) \end{bmatrix} = P(s) \begin{bmatrix} \Delta W(s) \\ \Delta U(s) \end{bmatrix}, \tag{2.10}$$

where $\Delta Z(s)$, $\Delta Y(s)$, $\Delta W(s)$ and $\Delta U(s)$ are the Laplace transforms of $\delta z(t)$, $\delta y(t)$, $\delta w(t)$ and $\delta u(t)$, respectively.

In this derivation, the back emf induced by the moving levitating magnet is ignored as it is very small. If the Hall effect sensor is located below the levitating magnet, then γ is also very small and it can also be neglected.

3. MODEL PARAMETERS

Letting the desired d_e be 2.00×10^{-2} m and using measurements, the parameters of the electromagnetic levitation system are determined as

$$\begin{array}{lll}
R = 2.41\ \Omega, & L = 15.03\times10^{-3}\ \mathrm{H}, & m = 3.02\times10^{-3}\ \mathrm{kg}, \\
g = 9.81\ \mathrm{m/s^2}, & k = 17.31\times10^{-9}\ \mathrm{kg\,m^5/s^2/A}, & \alpha = 2.48\ \mathrm{V}, \\
\beta = 2.92\times10^{-4}\ \mathrm{V\,m^2}, & \gamma = 0.48\ \mathrm{V/A}, & u_e = 0.66\ \mathrm{V}, \\
x_{1e} = 2.00\times10^{-2}\ \mathrm{m}, & x_{2e} = 0.00\ \mathrm{m/s}, & x_{3e} = 0.27\ \mathrm{A}, \\
z_e = 2.00\times10^{-2}\ \mathrm{m}, & y_e = 3.34\ \mathrm{V}, & w_e = 0.00\ \mathrm{V}.
\end{array} \tag{3.1}$$

With these parameter values, it follows that

$$\begin{array}{ll}
A = \begin{bmatrix} 0 & 1 & 0 \\ 1.9620\times10^3 & 0 & -35.8214 \\ 0 & 0 & -160.3460 \end{bmatrix}, & B_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0 \\ 0 \\ 66.5336 \end{bmatrix}, \\
C_1 = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}, & D_{11} = \begin{bmatrix} 0 \end{bmatrix}, \quad D_{12} = \begin{bmatrix} 0 \end{bmatrix}, \\
C_2 = \begin{bmatrix} -72.9965 & 0 & 0.4800 \end{bmatrix}, & D_{21} = \begin{bmatrix} 1 \end{bmatrix}, \quad D_{22} = \begin{bmatrix} 0 \end{bmatrix},
\end{array} \tag{3.2}$$

and

$$P(s) = \begin{bmatrix} 0 & \dfrac{-2.3833\times10^3}{(s+160.3460)(s^2-1.9620\times10^3)} \\ 1 & \dfrac{31.9361s^2+1.1132\times10^5}{(s+160.3460)(s^2-1.9620\times10^3)} \end{bmatrix}. \tag{3.3}$$

The noise process n can be modeled as a white noise with spectral height $N = 1.00\times10^{-9}$ $\mathrm{V^2\,s}$.

REFERENCES

[1] D. K. Cheng, *Field and Wave Electromagnetics*. MA: Addison-Wesley, 1983.

[2] A. Smaili and F. Mrad, *Applied Mechatronics*. MA: Oxford, 2008.